Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME										
CENTRE NUMBER						CANDIDATE NUMBER				

5019244541

FURTHER MATHEMATICS **9231/21**

Paper 2 Further Pure Mathematics 2 **May/June 2022**

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS
- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION
- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages.

1 The curve C has polar equation $r = e^{\frac{3}{4}\theta}$ for $0 \leqslant \theta \leqslant \alpha$.

Given that the length of C is s, find α in terms of s. [5]

2 **(a)** Starting from the definitions of cosh and sinh in terms of exponentials, prove that

$$\cosh 2x = 2\sinh^2 x + 1.$$ [3]

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(b) Find the set of values of k for which $\cosh 2x = k \sinh x$ has two distinct real roots. [5]

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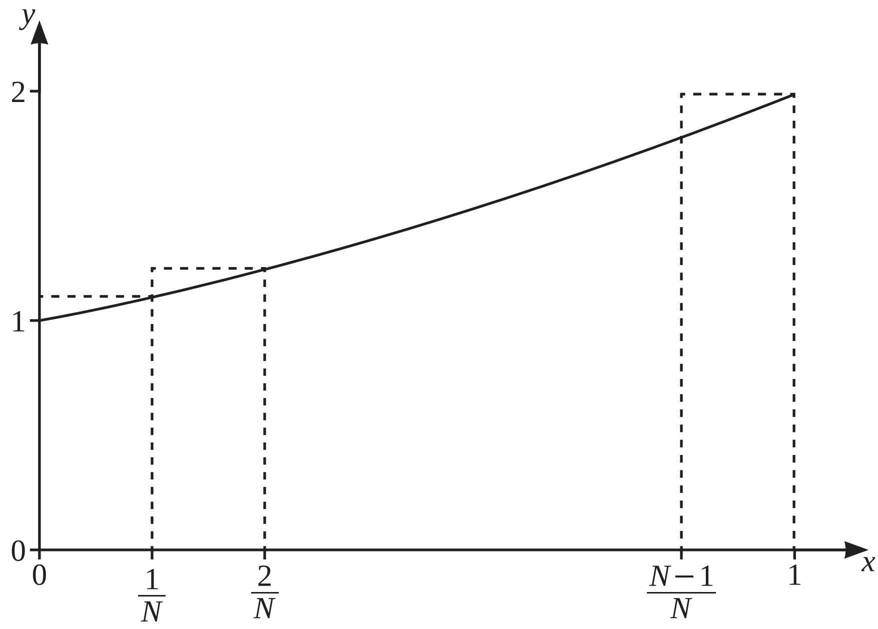
3 The variables t and x are related by the differential equation

$$\frac{d^2x}{dt^2} + \frac{dx}{dt} + x = t^2 + 1.$$

(a) Find the general solution for x in terms of t. [6]

(b) Deduce an approximate value of $\frac{d^2x}{dt^2}$ for large positive values of t. [2]

4 The diagram shows the curve with equation $y = 2^x$ for $0 \leqslant x \leqslant 1$, together with a set of N rectangles each of width $\frac{1}{N}$.



(a) By considering the sum of the areas of these rectangles, show that $\int_0^1 2^x \, dx < U_N$, where

$$U_N = \frac{2^{\frac{1}{N}}}{N\left(2^{\frac{1}{N}} - 1\right)}.$$

[4]

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(b) Use a similar method to find, in terms of N, a lower bound L_N for $\int_0^1 2^x \, dx$. [4]

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(c) Find the least value of N such that $U_N - L_N < 10^{-4}$. [2]

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5 The variables x and y are such that $y = 0$ when $x = 0$ and

$$(x+1)y + (x+y+1)^3 = 1.$$

(a) Show that $\frac{dy}{dx} = -\frac{3}{4}$ when $x = 0$. [3]

(b) Find the Maclaurin's series for y up to and including the term in x^2. [7]

6 Use the substitution $y = vx$ to find the solution of the differential equation

$$x\frac{dy}{dx} = y + \sqrt{9x^2 + y^2}$$

for which $y = 0$ when $x = 1$. Give your answer in the form $y = f(x)$, where $f(x)$ is a polynomial in x. [10]

7 **(a)** Use de Moivre's theorem to show that

$$\operatorname{cosec} 7\theta = \frac{\operatorname{cosec}^7\theta}{7\operatorname{cosec}^6\theta - 56\operatorname{cosec}^4\theta + 112\operatorname{cosec}^2\theta - 64}.$$

[6]

(b) Hence obtain the roots of the equation

$$x^7 - 14x^6 + 112x^4 - 224x^2 + 128 = 0$$

in the form $\operatorname{cosec} q\pi$, where q is rational. [5]

8 **(a)** Find the value of a for which the system of equations

$$\begin{aligned} 3x + ay \quad &= 0, \\ 5x - \ y \quad &= 0, \\ x + 3y + 2z &= 0, \end{aligned}$$

does not have a unique solution. [2]

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The matrix **A** is given by

$$\mathbf{A} = \begin{pmatrix} 3 & 0 & 0 \\ 5 & -1 & 0 \\ 1 & 3 & 2 \end{pmatrix}.$$

(b) Find a matrix **P** and a diagonal matrix **D** such that $\mathbf{A}^2 = \mathbf{PDP}^{-1}$. [7]

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(c) Use the characteristic equation of **A** to show that

$$(\mathbf{A}+6\mathbf{I})^2 = \mathbf{A}^4(\mathbf{A}+b\mathbf{I})^2,$$

where b is an integer to be determined. [4]

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.